QAM SIGNAL IMPAIRMENTS AND THEIR EFFECTS ON MER AND BER

Everything You Always Wanted to Know About MER But Were Afraid to Ask

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1. FOREWORD

This document is intended as a tutorial on the subject of MER and BER and the components that affect them.   It is designed for CATV communication engineers and senior technicians that already have a knowledge of Digital transmission on broadband networks and are familiar with QAM signals and their standard formats (ITU J83 Annex A, B and C).

The document specifically addresses the downstream conditions but the principles apply to the upstream as well, with appropriate scaling.

2. SCOPE OF THE DOCUMENT

The document defines the current terminology used in digital transmission and then draws the relationship between impairments to the digital signal and MER and BER.   The purpose is to provide an understanding of the effects of the impairments and their relative importance.

3. TERMINOLOGY

3.1. Source of definitions

The definitions of the terms used are extracted, as much as possible, from ETR 290 Digital Video Broadcasting, Measurement Guidelines for DVB systems[1].   This document is used because it has been generated by a recognized standards body and definitions are comprehensive and clear.
A definition may be followed by notes indicating its importance, relationship to other definitions, and/or comments.

All definitions that are a transcription of the ETR 290 standard are enclosed by <   >.

3.2. Modulation Error Ratio (MER)

MER <provides a single "figure of merit" analysis of received signals.>

< This figure is computed to include the total signal degradation likely to be present at the input of a commercial receiver's decision circuits and so give an indication of the ability of that receiver to correctly decode the signal.>

< The sum of the squares of the magnitudes of the ideal symbols vectors is divided by the sum of the squares of the symbol error vectors. The result is expressed as a power ratio in dB. >

Note: MER is the ratio of the power of the signal to the power of the error vectors.   It is interesting to note that we have a power of the error vectors or power of the impairments. Consequently, we will be able to separate the power contribution of the various impairments.
MER is sometimes referred to as Signal to Noise but this is not true even though Noise is usually the major contributor to signal impairment.

3.3. Error Vector Magnitude (EVM)

EVM is the same measurement as MER but expressed differently.   Not a power ratio but an amplitude ratio of the RMS error vector to the amplitude of the largest symbol, it is expressed in %.

Note: EVM gets worst as impairment increases.

3.4. Bit Error Rate before Forward Error Correction (BER Pre-FEC)

< The BER is defined as the ratio between erroneous bits and the total transmitted bits. >

Note: The Channel coding has a mechanism to correct a number of transmission errors (Reed-Solomon coding); in the BER Pre-FEC, the count of errors is made before this mechanism.   This is exactly what happens in the case of ITU J-83 Annex A and C.   In the case of Annex B, there is an additional error correction mechanism embedded within the constellation mapping, the Trellis coding.   As it is embedded, there is no access to errors before the trellis coding, the BER Pre-FEC is therefore after the trellis. This accounts for the "better" BER-Pre in Annex B than in A or C for the same MER.

3.5. Bit Error Rate after Forward Error Correction (BER Post-FEC)

The channel coding mechanism for correcting errors uses the Reed-Solomon (RS) coding.   This coding generates additional words that are used to correct some errors at reception.   If "x" words are added to a packet or block of data, RS codes will allow correction of x/2 errored words and detect almost all errors. Errors that cannot be corrected, do corrupt a packet that is then declared invalid and is scrapped by the user application.   With the size of packets used and the number of RS bytes added, the probability of not detecting errors is very small but is not 0.   In Annex A and C, the MPEG Transport Stream Reed-Solomon codes are used, that is 188 data bytes and 16 RS bytes; in Annex B it is 122 data words of 7 bits and 6 RS words of 7 bits.

In addition to RS error correction, "Interleaving" is used to spread the normally contiguous symbols over multiple blocks and then re-assemble them at reception.   If a burst of errors exceeds the capability of RS, it would normally mean a lost packet but the interleaved transmission spreads the burst over multiple blocks, yielding only a small error in each block. These small errors are now well within the RS's ability to correct, unless the burst of errors is so long that it exceeds the Interleave Burst Protection period.

3.6. System Availability

Corrected errors do not affect the user. On the other hand, a single uncorrected error invalidates a Block of data, that is declared:

< Errored Block (EB)　　　　　An MPEG –2 TS packet with an uncorrectable error, which is indicated by the transport_error_indicator flag. >

3.6.1. Errored Second (ES)

< Errored Second (ES)　　　　A one second period with one or more EBs >   i.e. at least 1 uncorrectable error or one Post –FEC error.

3.6.2. Severely Errored Second (SES)

< Severely Errored Second (SES)   A one second period which contains greater than a specified percentage of errored blocks, or at least SDP. This percentage will not be specified in this document, >

Note: The threshold for SES is programmable in the AT2000 between $10^{e^-}2$ and 10^{e}-4.   In J 83-B applications there are 16 000 to 25 000 packets / second, so for a threshold of 1e-3, there are 25 errored blocks per second.

3.6.3. Severely Disturbed Period (SDP)

< Severely Disturbed Period (SDP)　　　The duration of the Sync Loss (2 or more consecutive corrupted sync bytes of the transport stream) or loss of signal >.

Note: In the AT-2000, this is renamed to Frame Loss (FLs) because of the non-descriptive term.

3.6.4. Unavailable Time

< A period of Unavailable Time begins at the onset of 10 consecutive SES events. These 10 seconds are considered to be part of the Unavailable Time >

3.6.5. Estimated Noise Margin

< Estimated Noise Margin is computed by simulating the addition of white Gaussian noise to the demodulated data and predicting the resulting BER by statistical methods.

The noise margin will be the difference in dB between the estimated SNR of the received signal and the synthesized SNR, which gives a predicted BER of 10^{-4} (before RS decoding). >

Note: The calculation of ENM is complex and requires multiple iterations.   Also it is based on the assumption of noise as the only or very dominant factor. Finally, as developed for the DVB system, without Trellis coding, ENM is not as representative in Annex B conditions.
For all of these, we use ENM as the difference in dB between the measured MER and the minimum MER for 10e-8   BER Post-FEC as specified in DOCSIS Phy Layer (23.5 db and 30 dB for QAM 64 and 256) [2]. Experimental comparison between both indicated less than a 1 dB difference.

HISTORICAL NOTE: The ETR 290 standard was developed as a tool to characterize performance of a transmission channel, for common reference use by a signal supplier and a customer.   At the time this was developed some instrument makers developed different definitions for some terms as viewed from the inside of the transmission channel, such as "Errored second" defined as at least one error Pre-FEC [3].   But a BER Pre-FEC different from 0 gives the same information.

4. MODULATION ERROR RATIO COMPONENTS

With MER being a composite index of all QAM signal impairments, it is important to identify its various components.   Each of these components can be evaluated separately. For each, a "power" relative to the average signal power can be derived. Summing all these impairment powers we have the total impairment power that is used to calculate the MER.   This characteristic is interesting in that we can add them directly (in linear mode) because they can be considered uncorrelated. The "power" is the Mean Square of the Error Vectors for a statistically representative number of received symbols. Error Vector for a received symbol is the distance between the actual received symbol in the constellation to the ideal location of this symbol.
The power of a QAM signal is the Mean Square of the distances, from the centre of the constellation for all symbols in the constellation.   Hence, MER is the ratio in dB of the signal power to the Error Vector power.

Finally, lets not forget that an MER measurement is the composite measurement of 3 elements:
1) The Signal source, which is never perfect, has it's own MER;
2) The Network under test, which is usually what we want to characterize;
3) The Measurement instrument, which is also never perfect and has it's own MER, although it may be calibrated out.

4.1. Signal to Noise Ratio

Noise is the most common, and unavoidable, impairment to any signal including QAM. Additive White Gaussian Noise (AWGN) is the normal type of noise Impairment. As it is White (flat power density function in frequency) and Gaussian (mathematically "normal" amplitude density function), it spreads the received symbols in a cluster around the ideal location. A tri-dimensional representation density of error vectors is a "bell" with mathematically known function and thus a known probability of exceeding a set limit, i.e. crossing Symbol Boundaries a.k.a. Received error.

To generate a known S/N at the input of a receiver, we need to measure the power of the signal and the power of the added noise.

The power of a QAM signal can be measured by integrating the Power Density Function (PDF) over the bandwidth of the receiver. It can also be estimated, fairly accurately, by measuring the average power in a known bandwidth and extrapolating to the 3 dB BW (that is the Symbol rate, i.e. 5.0569 MHz or 5.3605 MHz in Annex B).

Similarly, Noise power can be measured by integrating it's PDF over the receiver bandwidth. It can also be estimated by measuring the average power, in a known bandwidth, and extrapolating to the 3 dB BW of the receiver.

The challenge is the receiver bandwidth. The RF / IF portion has a 3 dB (noise bandwidth) determined by the SAW filter (6 MHz in Annex B / C and 8 MHz for Annex A). The QAM demodulator also has filtering with the characteristics of Square Root Raised Cosine (the overall system has a Raised Cosine filter split in half between transmitter and receiver). The Square Root Raised Cosine has a noise or signal bandwidth equal to the Symbol Rate, the same bandwidth as the signal.

Thus by using the same measurement bandwidth (same filter) for signal and for noise, the S/N is the ratio of the average measurements.

So for a S/N of 35 dB, one can set a ratio of Signal Power Density to Noise Power Density of 35 dB.

The relative power of the noise (as given by the S/N) is one of the power components of MER.

Note: As it is so common, and usually the dominant element of MER, S/N or Estimated S/N is sometime used wrongly as MER.

4.2. Narrowband or CW Interference

The addition of a CW Interference to a QAM signal introduces an Error Vector that rotates as compared to the received symbols at the difference in frequency between the Carrier and the Interferer. The net result is the well-known "Doughnut" distortion of the constellation.
The amplitude of this interferer is the amplitude of the error vector, i.e. the radius of the doughnut.

The relative power of the interference (as given by the S/I, ratio of average signal power to interferer power) is one of the power components of MER.

There are many sources of Interference, some in the Modulator/Up-converter (residual Local Oscillator or beat), some in the distribution network (Ingress, distortion beats…) and also some in the receiving equipment (very dense tuners such as "single chip" are prone to beats between LO1 and LO2).

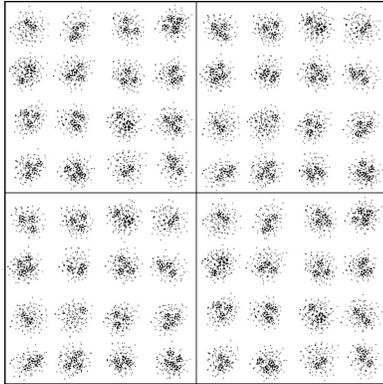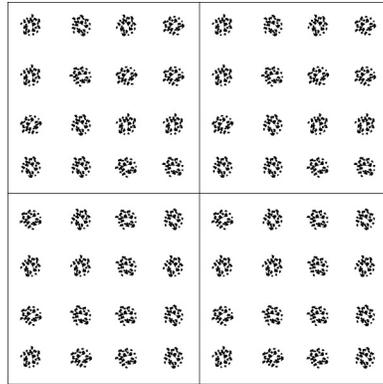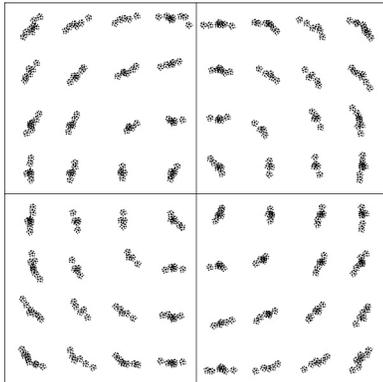**Fig. 1A   Noise**

Fig. 1B   CW Interference

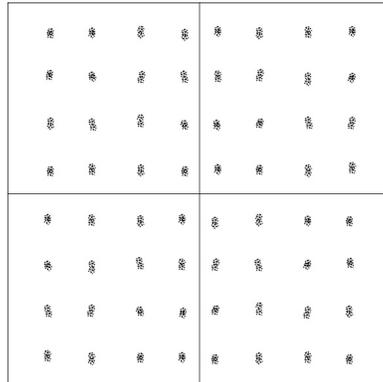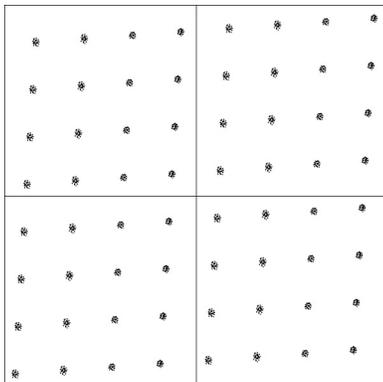**Fig. 1C   Phase Noise**

Fig. 1D   I / Q Gain Error

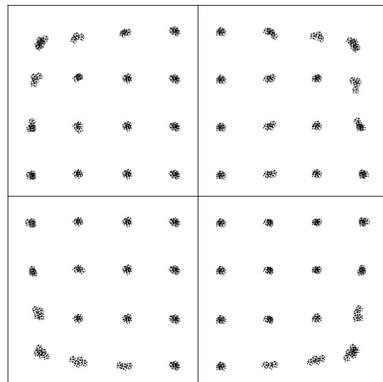**Fig. 1E   I / Q Phase Error**

Fig. 1F   Compression

TYPICAL IMPAIRMENTS ON A QAM 64 CONSTELLATION

FIGURE 1

NOTE ON CSO AND CTB

CSO and CTB are a narrow band interference but cannot be considered a CW interferer even though they look similar on a Spectrum Analyzer display.   To be measured, CSO and CTB are usually averaged to stand-out from the noise floor.   In reality they are widely varying in amplitude. These composite distortion products are the sum of many discrete beats of the various carriers in the system. Because of the small dispersion of the carriers frequencies, the beats are grouped in a narrow band. On a statistical basis, as the carrier phases align and drift apart, the composite beat increases and decreases in amplitude. Peak to average of 15 dB has been measured [14]. To a lesser extent, the amplitude of the carriers also increases and decreases with modulation levels, with peaks at the horizontal and vertical sync pulses that also drift slowly between different program sources [13].

The power of the CSO and CTB will affect the MER, as with other impairments, but the effect on the Constellation display will not be the familiar doughnut because of the varying amplitude. Rather, it looks like noise, especially in the presence of both a CTB and one or more CSO.

4.3. Phase Noise

Any Carrier Source or Local Oscillator in the signal chain has phase noise or phase jitter that is superimposed onto the received signal.   This phase noise is normally caused by thermal "agitation" within the oscillator, it is dependant on the noise of active devices (transistors), noise of passive devices (resistors) and shaped by the "Q" of the oscillating circuit.   It is defined for "normal" oscillators by the updated Leeson's Formula [4] [5].

$$L\,(fm) \;=\; \frac{F\,k\,T}{2\,P_o}\left[1+\left[\frac{f_0}{2\,Q_l\,f_m}\right]^2\right]\left[1+\frac{f_c}{f_m}\right]$$

Where L (fm) is the single side band noise at an offset frequency f_m;
F is noise figure, k is Boltzmann constant, T is temperature in Kelvin, P_o is oscillator power, f_0 is centre frequency, Q_l is the loaded Q and f_c is the Flicker noise.

This defines a Power Density Function for frequency offset relative to the Carrier frequency. There are 2 slopes to the PDF, $1/f$ (flicker noise), and $1/f^2$ (thermal noise).   The crossover point for the slopes is dependent on the characteristics of the oscillator.   It happens that the shape of PDF is modified by the use of a Phase Locked Loop circuit that minimizes low frequency variations, may enhance some mid-frequencies and does not affect high frequency offsets [6]. Flicker is usually a low frequency phenomena and is "removed" by the PLL, what is left in the band of interest is the $1/f^2$ component.

The relative Phase Noise power of the oscillator is the Integral of the PDF for both positive and negative offset frequencies (Double Sideband Noise).   For practical purposes the integral starts from a low value (10's or 100's of Hertz) up to the channel bandwidth (i.e. +/- 3 MHz for Annex B or C).

It is usually not convenient to do the integration of the PDF, so in many instances it is easier to do a Summation of phase noise power over 5 to 10 bands by taking typical values for each band (these bands are relatively narrow at low frequencies where the slope of decreasing PN is larger) [7].

A not-so-bad approximation can be done if the source is "well behaved and typical" (we assume only $1/f^2$ components) by measuring PN at the standard offset of 10 kHz and calculating the corresponding integrated Phase Noise.

This Phase Noise is cumulated from the signal source through the network to the receiving instrument; but the QAM demodulator has a PLL (or De-rotator) that tracks the incoming signal, this acts as a HIGH-PASS, reducing the PN at low offset frequencies but letting through high frequency offset PN.   Thus, the measured MER due to PN is the total PN multiplied by the amplitude response of the QAM De-rotator loop [7] [8].

The relative power of the Phase Noise (as given by the integral of the Phase Noise PDF multiplied by amplitude response of the QAM carrier tracking loop) is one of the power components of MER.

To emulate some known level of phase noise, it is convenient to phase modulate the signal source (or a Local Oscillator) with a known RMS value. This can be done with a sinusoidal modulation (RMS deviation = √2 pk deviation) at a modulating frequency far above De-rotator HP cut-off, such as 200 or 500 kHz. The relative Phase Noise power generated is that of the 2 sidebands created by the modulation, i.e. relative amplitude squared (for 1° rms deviation or 0.0174 radian RMS, phase noise power is $(0.0174)^2$,or 0.000 305 or –35.2 dBc).   1 radian = 57.3°, 1° = 0.0174 rad.

Example of Phase Noise calculation by Band Splitting and Summation for a "Typical" VCO with PLL and QAM demodulator 16 bands split, covers +/- 4.1 MHz or full Annex A channel — Phase Noise Reference taken at 10 kHz offset, extrapolated at ideal $1/f^2$ — PLL High-Pass response cut-off at 3.5 kHz, 12 dB / octave — QAM demod High-Pass response cutoff at 10 kHz, 12 dB / octave											
Band Split			VCO Phase Noise		VCO Noise Power in BW	PLL Resp.		Source Noise Power in BW	QAM dem. resp.		Effective Noise Power in BW
Centre Freq.	Start-Stop	Band-with	dBc/Hz	Linear		dB	Linear		dB	Linear	
2.25 kHz	2.0 / 2.5	0.5 kHz	-67	20^e-8	10^e-5	-10	0.1	1^e-5	-25	0.003	0.003^e-5
2.75	2.5 / 3.0	0.5	-69.8	13.2^e-8	6.6^e-5	-8	0.16	1.1^e-5	-22	0.006	0.006^e-5
3.5	3 / 4	1	-71	8.2^e-8	8.2^e-5	-6 dB	0.25	2.05^e-5	-18	0.016	0.03^e-5
4.5	4 / 5	1	-73	4.9^e-8	4.9^e-5	-5	0.32	1.6^e-5	-13	0.05	0.08^e-5
5.5	5 / 6	1	-74.8	3.3^e-8	3.3^e-5	-3	0.5	1.7^e-5	-10	0.1	0.17^e-5
7.0	6 / 8	2	-77	2^e-8	4.1^e-5	-2	0.63	2.6^e-5	-8	0.16	0.42^e-5
10.0	8 / 12	4	-80	1^e-8	4^e-5	-1	0.8	3.2^e-5	-6	0.25	0.8^e-5
16.0	12 / 20	8	-84	0.39^e-8	3.1^e-5	0	1	3.1^e-5	-3	0.5	1.6^e-5
28	20 / 36	16	-89	1.3^e-9	2.1^e-5	0	1	2.1^e-5	-1	1	2.1^e-5
52	36 / 68	32	-94	0.37^e-9	1.2^e-5	0	1	1.2^e-5	0	1	1.2^e-5
100	68 / 132	64	-100	1^e-10	0.64^e-5	0	1	0.64^e-5	0	1	0.64^e-5
196	132 / 260	128	-105.8	0.26^e-10	0.33^e-5	0	1	0.33^e-5	0	1	0.33^e-5
388	260 / 516	256	-112.2	6^e-12	1.5^e-6	0	1	1.5^e-6	0	1	1.5^e-6
0.772M	0.52/1.03	0.512M	-117.7	1.7^e-12	9^e-7	0	1	9^e-7	0	1	9^e-7
1540	1.03/2.05	1.024	-124	0.4^e-12	4.1^e-7	0	1	4.1^e-7	0	1	4.1^e-7
3076	2.05/4.1	2.048	-130	0.1^e-12	2.1^e-7	0	1	2.1^e-7	0	1	2.1^e-7
Total noise power of the VCO in one half of the full BW: 48.7^e-5						PLL noise half BW: 21^e-5			Demod noise half BW: 7.7^e-5		
Total noise power of the VCO both sidebands, full BW: 95.5^e-5						PLL noise full BW: 42^e-5			Demod noise full BW: 15.4^e-5		
Total noise power of the VCO, full BW in log scale: -30.1 dBc						PLL noise: -33.8 dBc			Demod noise: -38.1 dBc		
RMS phase noise angle: 0.31 rad or 1.79 deg						PLL RMS: 0.21 rad or 1.2 deg			Dem. RMS: 0.12 rad or 0.7 deg		

We note that more than half the noise power is removed by the synthesizer PLL and more than another half is removed also by the Demodulator PLL.

Table 1   Typical Phase Noise Calculation

4.4. Target Errors

While the previous impairments were dynamic, Target Errors group some impairments that are static, their values are dependent on a constant parameter.   This causes a displacement of the cluster of each constellation point, thus we can associate an error vector to each and calculate the total power of these vectors.

The source of target errors can be Gain or Phase errors between the In-Phase and Quadrature components or it can be compression.

The I/Q gain and phase errors are normally negligible in modern all-digital modulators. Such errors are not misalignment but rather equipment failure.   Compression on the other hand can be generated in modulators, up-converters and transmission network.

4.4.1. I / Q Gain error

A difference in Gain between the In-phase and Quadrature components of the constellation shapes it into a rectangle instead of a square. Each symbol is displaced toward one central axis and away from the other central axis, in proportion to their distances from the central axes. Hence there is an error vector in each axes for each symbol.

For a difference in gain of 4% (0.34 dB), there are 2 % in I and 2 % in Q.
Thus, to each symbol there is an associated 2% error vector (0.02). The relative power is (0.02)^2 = 0.000 4 or – 34 dB.
The average power due to gain error is 34 dB below the signal power.

The relative power due to I/Q gain difference is equal to (Delta G/100 / 2)^2 and is one of the power components of MER.   The factor of 100 converts from % to absolute value.

4.4.2. I / Q Phase error

An error in quadrature between the In-phase and Quadrature axes of the constellation shapes it into a lozenge or diamond instead of a square. Each symbol is displaced along the I and Q axes, in proportion to its distance from the central axes. Hence there is an error vector in each axes for each symbol.

For an I / Q phase error of 2° , each axes is shifted by 1° toward each other or away from each other.
An error of 1° causes a displacement or error vector of: tangent 1° = 0.017 or 1.7%.
(0.017)^2 = 0.000 3045 or –35.2 dB, the average power due to phase error is 35.2 dB below the signal power.

The relative power due to I/Q Quadrature Phase Error is equal to (Tangent <QPE / 2>)^2 and is one of the power components of MER.

4.4.3. Compression

Compression is a non-linear reduction in gain where symbols in the corners of the constellation (maximum signal amplitude) are affected, while other symbols closer to the centre of the constellation are little or not affected.
The non-linearity is a function of the circuitry in the network.   For a fair approximation, we suppose that only the corner symbols and the ones adjacent are affected; in QAM 64 there are 2 adjacent (¼ of the compression of the corner symbols), in QAM 256 the 2 closest adjacent at ½ the compression and the 3 next closest at ¼ the compression.

In QAM 64, for a 1% compression, there are 4 symbols at 1% compression or 0.0001 power, 8 symbols at 0.25% compression or 0.000 006 25, out of a total of 64 symbols, the averaged error power is 0.000 007 or –51.5 dB.
In QAM 256, for 1% compression, we have 4 symbols at 1% compression or 0.000 1 power, 8 symbols at 0.5% compression or 0.000 025 and 12 symbols at ¼ % compression or 0.000 006 25 out of a total of 256 symbols, the averaged error power is 0.000 0025 or –55.9 dB.

The relative power due to compression on the corner and adjacent symbols of the constellation is one of the power components of the MER. The exact power is dependant on the non-linearity function but can be approximated by:
QAM 64: (Compr x 1.1/100)^2 / 16, QAM 256: (Compr x 1.25/100)^2 / 64.
The factor of 100 converts from % to absolute value.

NOTE ON TARGET ERRORS: a factor of 2 in % or angle causes a change of 4 of the error power or 6 dB.

4.5. Example of MER Components Calculation

The following table gives an example of the calculations for MER components.

MER Components Calculation			
Component	**Value**	**Equation for impairment Power**	**Relative Power Ratio**
Gaussian Noise	S-N = 33 dB	10e-(S-N / 10)	0.000 501
Phase Noise	P-N = 0.8 ^{o} rms	(P-N * 0.0174)^2	0.000 195
CW Interferer	S-I = 38 dB	10e-(C-I / 10)	0.000 158
I / Q Gain	DG = 2 %	(DG/100 / 2)^2	0.000 025
I / Q Phase	QPE = 0.7 ^{o}	(Tangent <QPE / 2>)^2	0.000 149
Compression	Compr = 1 % (64 QAM)	(Compr x 1.1 /100)^2 / 16	0.000 008
Total Relative Impairment Power			0.001 036
MER as Ratio of Signal power to Impairment power expressed in dB			29.8 dB

Table 2 Example of MER Components Calculation
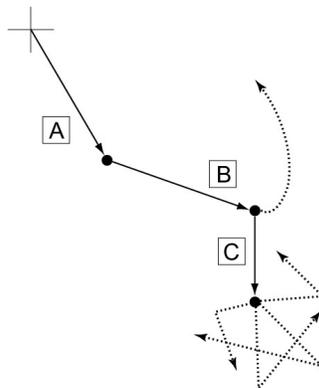
SUM OF 3 IMPAIRMENTS ON ONE CONSTELLATION SYMBOL
A is Static Target Error, B is Rotating CW Interferer, C is Random Noise

FIGURE 2

For the values of Impairment selected we can deduce that noise is the major contributor, but Phase noise + CW interferer + I / Q phase have an even greater impact.
With these selected Impairments we can also deduce that Gain components (I /Q gain and Compression have very little effect) it takes a catastrophic Impairment for these to impact MER seriously. On the other hand, Phase components (Phase noise and I / Q Phase) rapidly impact MER while Gaussian Noise and a CW Interferer have direct impact.

MER is a figure of merit for QAM signals but ultimately, for the end user, BER is the important factor. What is the relationship between MER and BER ?

5. BIT ERROR RATE VS MODULATION ERROR RATIO

5.1. Soft Decisions and MER

Bit Error Rate is another Figure of Merit in digital communication systems.   It indicates the rate of errored bits in a specific period, quite often a 1 second period, but it can be any specified period.

Errors occur when the incoming symbol being sampled has drifted from within its decision boundaries into another symbol (from one constellation limits to another).   In the demodulation process, the sampling of the incoming signal into symbols is called Soft Decisions.   At this point in time, no error correction mechanism has been called upon, it is the raw symbol.

There are 2 major error mechanisms, the statistical process and the impulse or burst errors.   The latter will be reviewed later.

Statistical errors occur because an impairment added to the desired signal is large enough to create a boundary crossing. The most common and unavoidable impairment is Gaussian Noise but all components of MER do contribute to spreading and/or shifting the "cluster" of the signal, statistically bringing it closer to the boundaries and increasing the probability of error.

There is a close relationship between Soft Decision errors and MER, it is not always simple but we will start with the Gaussian Noise because it is mathematically well defined.

5.2. Gaussian Noise and Probability of Demodulation Error

Additive White Gaussian Noise is representative of noise in a communication system, i.e. thermal noise as well as active devices noise.

It is mathematically defined as Gaussian if it has a "Normal" distribution of amplitudes, i.e.

$$f_X (x) \ = \ \frac{e^{-\,(\,x)^2\ /\ 2\,(\sigma)^2}}{\sigma \sqrt{2\pi}} \qquad [9]$$

where $f_X (x)$ is the probability that noise has an instantaneous value "x" and σ is the Root Mean Square of the noise, in other word σ^2 is the power of the noise. This is the "standard" bell shape distribution curve

Of real interest to us is the probability that the noise will exceed a given amplitude or threshold, such as the boundary to the adjacent Symbol i.e. the probability of an error at the demodulation stage.   This probability is as a function of the threshold expressed in multiples of σ.

Probability for AWGN to exceed a threshold expressed is multiple of σ							
Nb of σ	1	2	3	4	5	6	7
Prob.	$3.17^e\text{-}1$	$4.55^e\text{-}2$	$2.70^e\text{-}3$	$6.34^e\text{-}5$	$5.73^e\text{-}7$	$2.00^e\text{-}9$	$2.60^e\text{-}12$

Table 3 Probability of exceeding "x" Standard Deviation   [9]

**PROBABILITY OF GAUSSIAN NOISE TO EXCEED
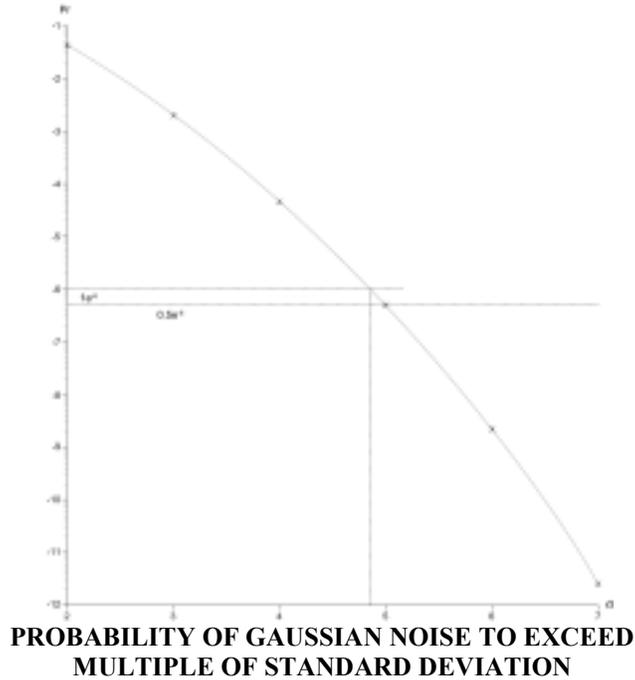MULTIPLE OF STANDARD DEVIATION**

FIGURE 3

This probability can be calculated relative to the ratio of "noise amplitude to the boundary distance" in multiple of σ (i.e. ratio of RMS amplitude of noise to the distance of the side of the Symbol decision box), and both of these can be related respectively to QAM 64 / 256 boundary spacing and S/N. This probability is "double sided". That is, AWGN instantaneous amplitude can be positive or negative.   For a "one sided" case, the probability is halved.

Constellation's 2 dimensions: The constellation, as a representation of the QAM signal has 2 dimensions, the I and Q axes, for signal as well as noise. If we consider the 4 corner symbols and assign them a normalized amplitude of "1", their I and Q components have an amplitude of 0.7071. But as noise is not correlated in the I and Q axes, we assign half the noise power to each axis.
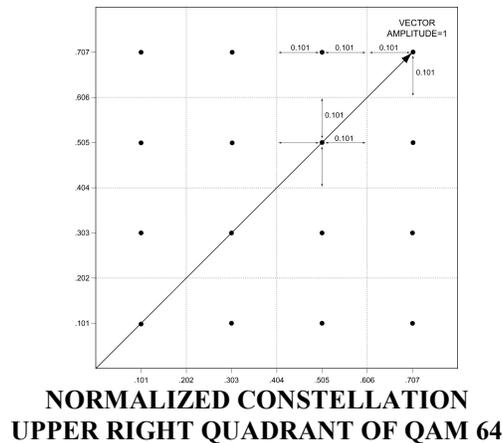

**NORMALIZED CONSTELLATION
UPPER RIGHT QUADRANT OF QAM 64**

FIGURE 4

Boundary spacing: For an ideal signal that has a maximum symbol amplitude of "1" (corners of the constellation), the maximum symbol amplitude in the I and Q axes is +/- 0.7071.
For QAM 64, from +0.7071 to –0.7071 there are 8 symbols, 7 boundaries and the distance from the ideal Symbol location to the closest boundary is 2 x (0.7071) / 2 = 1.4142 / 14 = 0.1010.
For QAM 256, the same range of +0.7071 to – 0.7071 has 15 boundaries and the distance from the ideal to the adjacent boundary is 1.4142 / 30 = 0.04714, or less than half of the QAM64 distance.

Signal to noise: For an ideal signal that has a maximum symbol amplitude of "1" (corners of the constellation), the average power is 3.7 and 4.2 dB (QAM 64 and 256) lower that the power of that maximum symbol.
Hence, for a 35 dB S/N (as an example), the half noise power is -35 –3 –3.7 (or 4.2) = -41.7 (42.2) dB below the power of the maximum symbol.

For QAM 64, boundary distance is 0.1010, at S/N of 35 dB, the –41.7 dB means 0.0082 RMS amplitude of noise, the distance in number of σ = 0.1010 / 0.0082 = 12.3; from this we can calculate the probability of exceeding the boundary i.e. the probability of a demodulation error $<<<2.6^e$-12. Because of noise in both axes, the probability of demodulation error is twice $<<<2.6^e$-12 !
For the same 35 dB S/N, in QAM 256, the boundary distance is 0.04714, the half noise power is –42.2 dB or 0.00776 and the distance in number of σ = 0.04714 / 0.00776 = 6.07, the probability of demodulation error is now approximately 2 x 2^e-9 or 4^e-9, i.e. one error every 6 seconds at 40 Mbits/s. This is for a perfect signal and demodulator with only Additive White Gaussian Noise. Other degradations will increase the probability of error.

S / N and Demodulation Error Probability						
Signal to Noise	**QAM 64, relative boundary distance = 0.1010**			**QAM 256, relative boundary distance = 0.04714**		
	Noise σ	**Nb of σ**	**Probability of error**	**Noise σ**	**Nb of σ**	**Probability of error**
38 dB	0.0058	17	$<<<2^e$-12	0.0055	8.5	$<2^e$-12
36 dB	0.0073	14	$<<<2^e$-12	0.0069	6.8	6^e-12
34 dB	0.0092	11	$<<<2^e$-12	0.0087	5.4	8^e-8
32 dB	0.0116	8.7	$<<<2^e$-12	0.0110	4.3	1^e-5
30 dB	0.0146	6.9	4^e-12	0.0138	3.4	6^e-4
28 dB	0.0184	5.5	8^e-8	0.0174	2.7	1^e-2
26 dB	0.0232	4.4	2^e-5	0.0220	2.1	1^e-1
24 dB	0.0292	3.5	2^e-2	0.0275	1.7	0.5^e0

Table 4 Signal to Noise and Demodulation Error Probability

5.3. Channel Coding and BER

In order to enhance the performance of a communication channel, Forward Error Correction channel coding is used to correct a fair number of demodulation errors. The most common FEC is Reed-Solomon coding that calculates Check codes appended to the data packet at the source. After demodulation and recalculation, at reception, any discrepancy indicates an error in transmission. Furthermore, words in error can be corrected up to 1 correction for every 2 check code words.

J83 Reed-Solomon Forward Error Correction						
Annex	**Data packet**		**R-S Code words**		**Correctable Words**	**RS as % of packet**
	Nb of words	**Nb of bits**	**Nb of words**	**Nb of bits**		
B	122	7	6	7	3	4.7 %
A, C	188	8	16	8	8	7.8 %

Table 5 FEC Coding Formats

J83 Annex B has an additional powerful channel coding: Trellis coding.
Trellis coding is embedded in the modulation process so it often carries the name Trellis Coded Modulation.

Trellis Coded Modulation At the signal source, Trellis coding is twofold: an encoder that selects an optimal sequence from the data and a mapping of the constellation. The constellation in TCM is larger than normally required

for the data words and the encoder generates a sequence of wider words mapped such that the transition distances in the constellation are maximized.

The purpose of increasing the constellation size is to implement rules of coding where transitions from a constellation point are allowed to only some specific other points (all combinations are not allowed) depending on the previous data. The objective being to increase the distance of transitions over that sequence.

At reception, the TCM decoder calculates the most probable sequence that had been transmitted, even though it had been corrupted, using the same coding rules in selecting paths for the sequence. All possible paths are considered for likelihood and the most probable is chosen [10].   Trellis Coded Modulation does reduce the probability of error for a given S /N or, in other words, for a given Error Rate the S / N can be reduced by "x" dB, known as the coding gain.

The TCM coding gain as used in J83 Annex B is 4.47 and 4.55 dB respectively for QAM 64 and 256 [12].   As decisions on the value of the data are probabilistic and embedded in the demodulation process we cannot have a Pre-TCM bit error rate.

This TCM explains the much better Pre-FEC BER in Annex B as compared to Annex A or C for the same conditions. It also justifies the lesser performing FEC in Annex B.

5.4. BER vs MER components

The previous analysis has been based on error probabilities (crossing of constellation boundaries) due to Additive White Gaussian Noise; but there are more channel impairments than noise. We have previously quantified the impairments in the MER components calculations, now we will analyze the effect of the impairments on BER.

In addition to the AWGN, 3 types of impairment will be considered: target errors, CW interference and phase noise.

Target Errors These are static or quasi-static displacement of the "noisy cluster" in the constellation, due to I / Q gain or I / Q phase errors or to Compression. As seen previously, these affect mostly the periphery of the constellation and less toward the centre.   The effect on demodulation errors is an increase in probability of errors because it reduces the effective distance to the boundary, the ratio of effective distance in σ is smaller on one or 2 sides (it is increased on the others, but the reduction in probability of errors does not compensate for the increase on the first sides).
The overall increase in errors is weighted by the number of constellation points for each value of target errors.

CW Interference It is somewhat similar to target error in that the "noisy cluster" is displaced, but in a dynamic way. The displacement rotates around the ideal symbol location, at a fixed radius and it affects all constellations points equally.

Phase Noise Again this is a displacement of the "noisy cluster", but the conditions are more complex. As for target errors, it affects mostly the periphery of the constellation so we need to weight its effect on demodulation errors. The main difference it that phase noise is highly dynamic, in most cases it is noise with a defined spectral density (pink noise) and a specific amplitude distribution function (it could be quite different from the normal Gaussian distribution) so it requires a good knowledge of this distribution to calculate the probabilistic effect on demodulation errors.   But most usually it is Gaussian [7].

BER degradation calculation example These calculations are done before any FEC or TCM and is more appropriately named "demodulation errors".
As an example of Target error, I / Q gain error of 2% (as seen in 5.4.1), gives a +/-1% error in both I and Q.
Lets take QAM 64 and 30 dB SNR hence σ = 0.0146, Nb of σ = 6.9 (see para 6.2).   The effect is most noticeable on the outer symbols of the constellation, 30 symbols out of 64. The relative boundary distance is 0.1010 so a 1% displacement is 0.01; the relative distance to the boundary is no more 0.1010 / 0.0146 = 6.9 but (0.1010 – 0.01) / 0.0146 = 6.2,   the probability of error increases from 4^e-12 to 5^e-10 (divided by 30 /64 most affected symbols) that is approximately 2^e-10 an increase of 2 orders of magnitude !

Because of the highly non-linear probability of error vs. number of σ, the increase in BER for a given impairment (but noise) is very dependant on noise, the lower S/N the lesser effect. At 26 dB S/N and the same I/Q gain error of 2%, the probability of error increases 2.5e-5 to 5e-5.

5.5. Interleave and Impulse Noise

Interleave is a mechanism where data packet with FEC are not transmitted as formatted but rather transmitted by inter-spacing the content of consecutive packets. At reception, the received bits are de-interleaved to construct the original packets. The purpose is to spread burst errors in transmission where a large number of adjacent bits are errored. Without interleave, many adjacent errors cannot be corrected. With Interleaving, having spread the burst of errors into a few errors per packet but over many packets, FEC is able to correct these few errors in each packet.

Interleave will transmit the first word of the current packet, then first word of the previous packet , then the first word of the second previous packet… until all first words of all packets are transmitted. It then starts again with the second words of the set of packets and then the third….[11].
Interleave are defined by 2 parameters I and J. I x J = packet size or multiple of packet size.
In Annex B, the packet size is 128 (122 data words and 6 FEC words), so the available Interleaves are
I = 128 x J = 1, or I = 64 x J = 2, or I = 32 x J = 4…. or the enhanced interleave: I = 128 x J = 2, or I = 128 x J = 3…
up to I = 128 x J = 8.
In I =128 x J = 1, there is a set of 128 packets.
In I = 128 x J = n, there are "n" sets of 128 packets.
In I = m x J = n, there are "n"sets of "m" packets such that m x n = 128.

In Annex A / C, packet sizes are 204 words, and the interleaved is fixed at I = 12 x J = 17.

There are 2 major consequences to Interleave. First, there is the Burst Protection Period, that is the duration of a burst where all contiguous errors in transmission could be corrected with FEC. Second is Latency, which is a delay introduced into the channel and is given by the period required to interleave then de-interleave one full packet. It is long enough that DOCSIS has limited interleave to I = 128 x J = 1, while Digital video uses up to I = 128 x J = 4 taking advantage of the longer Burst Protection Period.

INTERLEAVE CHARACTERISTICS					
I	**J**	**QAM 64**		**QAM 256**	
		BURST PROTECTION	**LATENCY**	**BURST PROTECTION**	**LATENCY**
128	**1**	**95 μsec**	**4 msec**	**66 μsec**	**2.8 msec**
64	**2**	47	2	33	1.4
32	**4**	24	0.98	16	0.68
16	**8**	12	0.48	8.2	0.33
8	**16**	5.9	0.22	4.1	0.15
128	**1**	95	4	66	2.8
128 *	**2**	190	8	132	5.6
128 *	**3**	285	12	198	8.4
128 *	**4**	379	16	264	11
128 *+	**5**	474	20	330	14
128 *+	**6**	569	24	396	17
128 *+	**7**	664	28	462	19
128 *+	**8**	759	32	528	22
12 #	**17**	18	0.156	14	0.116
12 ##	**17**	23	0.196	18	0.147

Table 6 Interleave Burst Protection Period and Latency

Notes:
*   Not supported by DOCSIS
+   Not currently in use by Digital video
\#   Annex A at 6.9 Ms/s
\##   Annex C at 5.2 Ms/s

Note on CSO and CTB: As we have seen in the MER components analysis, the CSO and CTB vary widely in amplitude, up to 15 dB peak to average ratio [13] [14]. Hence, they affect BER as burst errors, taxing the de-interleave and FEC decoding. As a narrow band composite of many stable carriers, beat bursts are relatively long, in the 50 to 200 μseconds [14] [15], that is the inverse of the carrier frequency offset dispersion These beat bursts are quite different from channel noise bursts that may have a similar peak to average ratio but have much shorter peak duration, well under de-interleaver protection period. Consequently, CTB / CSO induced bursts exceeding the interleave protection period generate a larger number of consecutive errored blocks.

5.6. Equalizer Operation

An equalizer is an adaptive filter whose function is to "optimize" the received signal before demodulation. The original purpose of equalizer is to minimize Inter-Symbol Interference, that is the residual of one symbol unto its neighbors due to the filters in the communication channel as well as reflections. It is a filter whose amplitude and phase responses are the inverse to that of the communication channel, such that the overall response restores the original signal.

To achieve this in digital form, one uses a Time Filter where samples of the signal (at symbol sampling time) for the current symbol and for previous and future samples are processed. The samples are fed into delay elements or taps; a reference element is chosen as the current symbol position so previous symbols are behind and future symbols are ahead.
The filtering function is accomplished by feedback from these other symbols unto the reference symbol through variable gain and phase control. By varying the complex coefficients (gain and phase) of each tap multiplier, the response of the filter is adapted to the channel [10].

In a Cable Digital equalizer it is common to have a set of 7 Feed Forward Equalizer taps (plus the current symbol as the 8th element of the delay line) and 24 "Decision Feedback Equalizer" taps . The ensemble of the 32 sets of coefficients generates the "Time Impulse Response" of the filter (the 8th reference tap coefficient has a nominal gain of 1 and zero phase) [11].
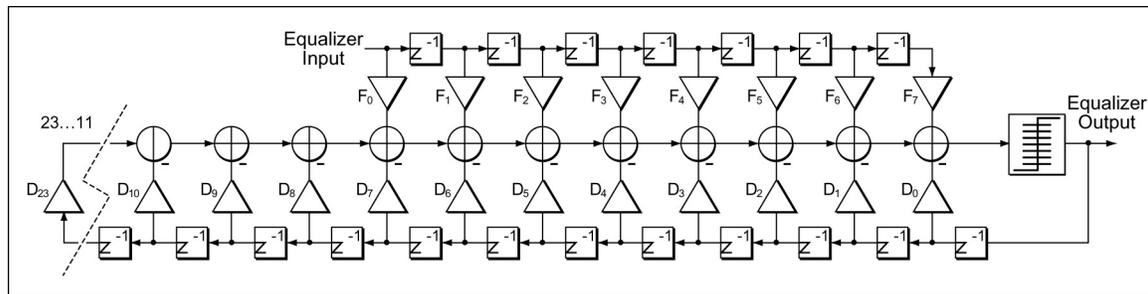
TYPICAL EQUALIZER FOR J-83 ANNEX B DEMODULATOR
F0 to F6 are FEEDFORWARD TAPS, F7 IS THE REFERENCE TAP
D23 to D0 are DECISION FEEDBACK TAPS

FIGURE 5

In order to adapt the filter coefficient to converge toward the "ideal" signal, the Decision Feedback Equalizer uses the symbol after decision, that is the ideal value for this symbol, instead of its "analog noisy" value.
Adaptation of the filter coefficient is a critical element in the performance of a digital signal receiver. Some communications systems use training sequences where a known data pattern is sent prior to the actual data to allow coefficient calculations. This is used in the Upstream channel where short bursty transmission does not allow long adaptation period. In the downstream continuous transmission, "blind" equalization is performed, coefficients are modified in small increments, if it improves reception it keeps on the same track; if it degrades, attempts are made in other directions (gain / phase) in order to converge toward the optimum. Too large increments may prevent convergence by jumping over the "sweet spot", too small increments make the convergence too long.

But how is "improved reception" defined? It is defined by the "Mean Squared Error", that is the calculation of the MSE between the received symbols and their ideal value. This MSE is a power, the impairment power, in other words the **Equalizer adapts the filter to optimize the MER** ! Various convergence methods are available including the Least Mean Square algorithm.

The filter optimizes the MER but it cannot compensate for everything, it cannot remove random processes such as noise or phase noise, it cannot compensate non-linear distortion such as compression or I /Q gain or phase error (the equalizer may attempt to compensate those and may "hunt" between 2 conditions). Note: other types of equalizing mechanism can compensate for I/Q gain and phase errors, but are not used in CATV applications.

There is another type of impairment that the Equalizer can alleviate. The Equalizer can reduce the effect of CW interferer if the latter is programmed for. CW interference shows up as a rotating vector added to the QAM signal. If high enough, it is visible in the doughnut shaped constellation. As this rotating vector is constantly changing in phase, from one symbol to the other, it is difficult for a fast acting equalizer to act on it. But if the equalizer convergence factor is adjusted for and integrated over a very large period (tens of millions of symbols, 5 / 6 seconds at 5 Ms/s) the equalizer generates a "notch" in the frequency response to reduce the CW interference.
Not all equalizers in current CM or set-top boxes have this feature that has been introduced to alleviate problems when "single chip tuners" are used. In those, the inevitable proximity of the 2 Local Oscillators creates some beats or CW interferer at some tuning frequency.

The capability of an Equalizer to correct for some Impairments is due to their predictability, on the other hand the random nature of noise prevent its reduction by Equalization.

The Equalizer is a time filter, so the set of coefficients for the sequence of taps is effectively the Impulse Response of the filter. Consequently, a Fourier Transform gives the amplitude and phase responses of the filter.

6. CONCLUSION

Digital Transmission for Cable Modems and Digital Video, using the J-83 formats, is a robust channel-coding scheme, especially the Annex B.

This allows reliable Downstream transmission with a high bit rate for a given channel bandwidth, despite all impairments in the system.

Upstream is based on the same fundamental principles, but the poorer channel conditions force the use of lower modulation ratios (QPSK or QAM 16) or Spread Spectrum techniques.

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